Notes: Ratios

"Recently it was reported that the Director-General of the Public Service Department stated that the government is working towards providing better allocation of teachers in schools. Currently the ratio is 1 teacher for every 40 students. The target is to allocate one teacher to every 20 students."

Source: http://www.malaysiatoday.com/

In the paragraph above you see the word 'ratio' being used. What is a ***ratio***? Simply put, a ratio is a comparison of two numbers or things. By saying that the ratio is 1 teacher for every 40 students, we are comparing the number of teachers against the number of students.

How many teachers are there? 1.
How many students are there? 40.

In math, a ratio is always written in the form of a fraction. If you know how to deal with fractions, you can definitely work with ratios as well. To write a ratio as a fraction, the first number becomes the numerator, and the second number in the ratio becomes the denominator. So in symbols we have the following:

Given a and b are any two numbers, the ratio of a to b is:

$$\frac{a}{b} \quad (\text{where } b \neq 0)$$

Example 1: Write the ratio of 15 boys to 10 girls as a fraction.

Solution: The number of boys is mentioned first, and so the number of boys will be in the numerator.

The ratio of 15 boys to 10 girls can then be written as:

$$\frac{15}{10}$$

As you can see the fraction above can be reduced. And so we have:

$$\frac{15}{10} = \frac{3}{2}$$

From the reduced fraction above, we can also say that the ratio of 15 to 10 is the same as the ratio of 3 to 2.

Example 2: Express the ratio of 36 to 42 as a fraction in lowest terms.

Solution: The number 36 appears first, so it will be in the numerator.

$$\frac{36}{42} = \frac{6}{7}$$

Therefore the ratio of 36 to 42 can be written as the fraction $\frac{6}{7}$.

The numbers involved in a ratio may **not necessarily** be **whole numbers or integers.** Look at the following example.

Example 3: Express the ratio of $\frac{2}{5}$ to $\frac{4}{3}$ as a fraction in lowest terms.

Solution: The fraction $\frac{2}{5}$ appears first, so it will be in the numerator.

$$\frac{\frac{2}{5}}{\frac{4}{3}} = \frac{2}{5} \div \frac{4}{3}$$

We have yet to simplify our answer. Do you remember how to divide fractions?

When we divide fractions, we change the operation to multiplication like so:

$$\frac{\frac{2}{5}}{\frac{4}{3}} = \frac{2}{5} \div \frac{4}{3} = \frac{2}{5} \times \frac{3}{4}$$

And so we have

$$\frac{2}{5} \times \frac{3}{4} = \frac{1}{5} \times \frac{3}{2} = \frac{3}{10}$$

Therefore the ratio of $\frac{2}{5}$ to $\frac{4}{3}$ can be written as the fraction $\frac{3}{10}$.